Dynamics and Control of Orbiting Flexible Structures Exposed to Solar Radiation

R. Krishna* and P.M. Bainum†
Howard University, Washington, D.C.

Expressions for thermal deflections of uniform thin beams and plates exposed to solar heating are obtained as a function of the properties of the material and solar incidence angle. The major effect of the solar radiation pressure interacting with the thermally deformed structure is found to give rise to disturbance moments on the structure. The thermal deformations of the structures are assumed to be within 0.1% of the characteristic length of the structure. With the assumed thermal deformations, the resulting uncontrolled transient responses of these geosynchronous orbiting structures to the solar radiation pressure induced disturbances are simulated. The resulting rigid modal oscillations are found to be an order of magnitude larger than for those cases previously considered in which only the solar radiation pressure effect on vibrating structures was treated. Modifications of control laws and/or the feedback gain values are considered in order to improve the transient response characteristics under the thermally induced disturbances.

Introduction

THE major environmental disturbances on proposed orbiting large space structural systems are expected to be due to solar radiation pressure and solar heating. The dynamics and control of a flexible beam and a flexible plate in the presence of disturbances due to the solar radiation pressure acting on the vibrating structure were considered previously.[1-3] It was seen that the major effect of the solar radiation pressure interacting with an elastically deformed (vibrating) orbiting structure was to produce moments on the structure resulting primarily in rigid modal oscillations. For the case of extremely flexible structures the amplitudes of these modes may be appreciable, even in the presence of both active and passive control.[1-3] In some situations the control laws previously developed by ignoring environmental effects may have to be redesigned. For simple low order systems the feedback gain values may be suitably adjusted; however, for large order systems the versatility of the linear Gaussian technique can be used to redesign control laws which provide a compromise between transient performance and the required control effort.[1-3]

An important environmental effect is heat due to solar radiation that results in thermal gradients in a structure. The deformations caused by the thermal gradients can be very large and can result in the dynamic instability of a structure.[4-6] Another form of environmental disturbance is when solar radiation pressure interacts with a thermally deformed structure. The deformations caused by solar heating depend on the thermal properties of the material and the geometric shape of a structure. The selection of materials with desired thermal properties and the careful selection of a structural design are required to minimize the thermal deformations of a structure to an acceptable level. The thermal deformations of a structure will occur as long as the structure is in a sunlit orbit. Furthermore, the continuous removal of this deformation by using active control may not be practical. The thermal deformations will have to be minimized by careful consideration of the thermal properties of the material in the preliminary structural design process. The objective of the present paper is to consider the effect of solar radiation pressure on structural beams and plates which undergo thermal deflections due to solar heating. (To the author's knowledge, this is the first attempt to incorporate such effects into simulations that model the dynamics of large flexible orbiting systems.) Motions of beams and plates about a local vertical orientation and a local horizontal nominal orientation (the latter carrying a rigid gimballed dumbbell to provide gravity stabilization) will be considered for this study (Figs. 1 and 2).

Expressions for the thermal deflections of beams and plates exposed to solar heating will be developed. Subsequently, a mathematical model for the solar radiation induced torque on thermally deflected structures will be obtained. The uncontrolled and controlled dynamics of orbiting structures will then be simulated by considering the combined effect of the solar radiation pressure and solar radiation heating on vibrating structures. Modifications of the control law and the feedback gain values which are required to control the shape and orientation of the structure will be proposed.

In this study, the statically induced thermal deflections are assumed to be small relative to the characteristic structural dimensions. In addition, other major assumptions made here are 1) the reflection of solar radiation by the Earth (albedo) can be neglected; 2) the inherent time lags in the heat transfer process are very small compared with the orbital period and are ignored; 3) the radiation from the edge surfaces can be neglected; and, 4) the temperature distribution and the thickness of the beams and plates are uniform. The effects of the Earth's shadow and local shadowing due to another part of the structure are not included in the study.

Equilibrium Temperatures of Thin Plates and Beams

The cross section of a thin plate (or a beam) exposed to heat from solar radiation is shown in Fig. 3. The solar incidence angle θ_i is taken to be a constant during a small interval of time. During this interval, the surface facing the sun, s_u, attains a temperature T_1, and the surface away from the sun, s_l, attains a temperature T_2. The equilibrium temperatures T_1 and T_2 can be determined by writing the thermal energy balance equations. The total heat leaving the beam from the two surfaces s_u and s_l should be equal to the heat received by

Received Oct. 16, 1984; revision received Feb. 21, 1985.

*Senior Graduate Research Assistant, Department of Mechanical Engineering. Student Member AIAA.

†Professor of Aerospace Engineering, Department of Mechanical Engineering. Associate Fellow AIAA.

the beam.[7] Therefore,

$$E_1\sigma T_1^4 + E_2\sigma T_2^4 = \alpha_s G\cos\theta_i \tag{1}$$

where E_1 and E_2 are the emissivities of the surfaces s_u and s_l, respectively. σ is the Stefan-Boltzman constant $= 56.7\times 10^{-12}$ $kw/m^2 - K^4$; α_s is the absorptivity of the surface, s_u; and G the intensity of solar radiation $\simeq 0.8$ kW/m^2.

The heat flowing through the plate at equilibrium is also equal to the heat radiated from the surface s_l.[7]

$$E_2\sigma T_2^4 = k(T_1 - T_2)/t_c \tag{2}$$

where k is the thermal conductivity (kW/m-K) of the plate material, and t_c the thickness of the plate. Equations (1) and (2) can be rearranged as

$$T_1 = T_2 + (E_2\sigma t_c/k)/T_2^4 \tag{3}$$

$$T_2^4 = (\alpha_s G\cos\theta_i)/E_2\sigma - (E_1/E_2)T_1^4 \tag{4}$$

Equations (3) and (4) can now be solved to obtain T_1 and T_2 by assuming an approximate value of T_1 or T_2 and then applying numerical iteration. Assuming $E_1 = E_2 = 0.05$ and $\alpha_s = 0.2$ (characteristic of the proposed supporting mast material for large space structural systems), the temperature difference, $\Delta T = T_1 - T_2$, is obtained as a function of the solar incidence angle θ_i and various ratios of the parameter, $k_r = k/t_c$, as shown in Fig. 4. A higher value of k_r indicates a larger thermal conductivity value, and, therefore, the temperature difference between the two surfaces becomes smaller. A plate of thickness 1 cm and made of polyamide ($k = 0.25\times 10^{-3}$ kW/m-K) will have a maximum temperature difference of 2.3 K. The temperature gradient is found to vary approximately and proportionally to $\cos\theta_i$ (Fig. 4). Expressions for deflections of the plate which are a function of the temperature gradient are developed in the next section.

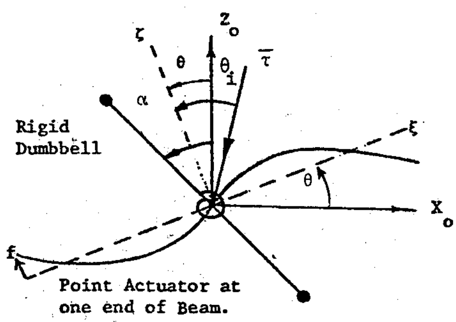

Fig. 1a Dumbbell-stabilized flexible beam nominally oriented along the local horizontal.

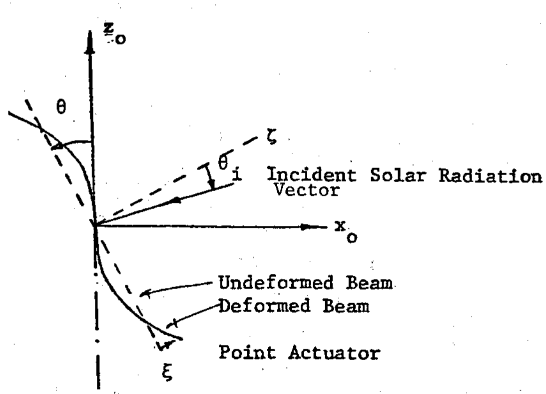

Fig. 1b Uniform flexible beam nominally oriented along the local vertical.

Pure Bending of Thin Plates and Beams

Figure 5 shows a beam of length ℓ and width b. The temperature of the mid-plane of the beam is denoted by T_n. The temperature of the surface facing the sun, s_u, is then $T_n + (\Delta T/2)$, and the temperature of the surface, s_l, is $T_n - (\Delta T/2)$. According to the theory of beam bending analyzed in Ref. 8, we have

$$\frac{d^2z}{dx^2} = -(\alpha_e/J_y)\int Ty\,dA \tag{5}$$

where z is the transverse deflection of the beam; α_e is the coefficient of linear expansion; and J_y the moment of inertia of the beam about the y axis. Equation (5) is rewritten by evaluating the integral so that

$$\frac{d^2z}{dx^2} = -\alpha_e(\Delta T/t_c) = \text{a constant} \tag{6}$$

The expression for the thermal deflection is then given by

$$z = -\alpha_e(\Delta T/t_c)x^2/2 \tag{7}$$

The thermal deflection can be minimized by selecting a material with a low coefficient of expansion or by using a material of high thermal conductivity. An increase in the thickness of the plate will increase the temperature difference (Fig. 4) and also increase the weight of the plate. Hence, the

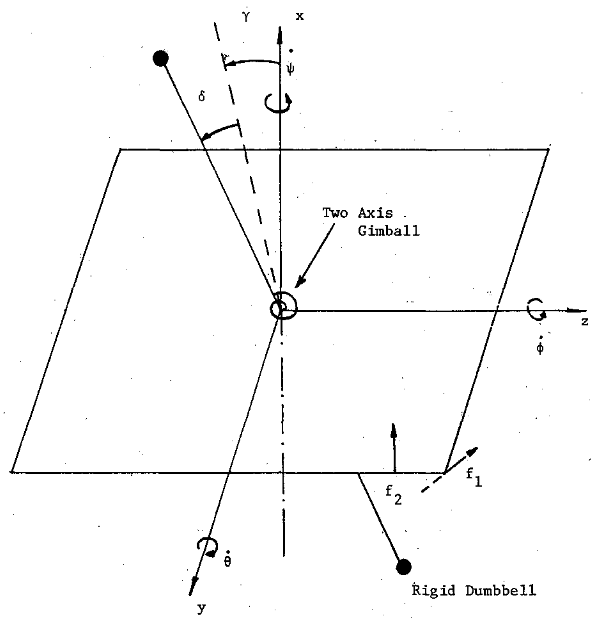

Fig. 2a Dumbbell-stabilized plate in orbit.

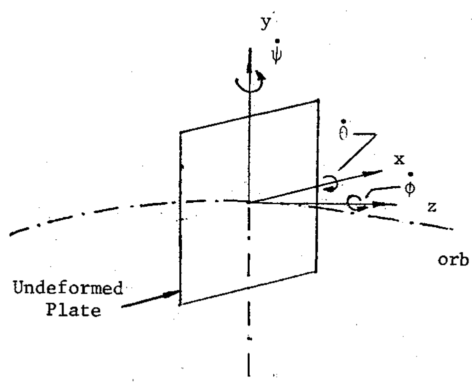

Fig. 2b Plate in orbit nominally oriented along the local vertical.

parameter t_c should be as small as possible. The other important properties of materials not reflected in Eq. (7) are the density and cost of the material as shown in Table 1.[9] For a beam made of polyamide (a low density and low cost material) of length 100 m and thickness 0.01 m, the maximum thermal deflection is found to be approximately 7 m. If the beam is made of aluminum, the maximum deflection would be about 2 mm.

Once a tolerable thermal deflection is specified the material can be selected to meet the conflicting requirements of low density, high thermal conductivity, and low cost. In the next section the solar radiation pressure moment that results from thermal deflection of a beam (also applicable to a plate) is discussed.

Effect of Solar Radiation Pressure on Beams and Plates Deformed Due to Solar Heating

The moment expressions obtained by Karymov[10] are used here to develop the solar radiation disturbance model for thermal deflection of beams and plates. The solar radiation moments acting on a completely absorbing surface, $\bar{N}_a$, and a completely reflecting surface, $\bar{N}_r$, are given by[10]

$$\bar{N}_a = h_0 \bar{\tau} \times \int_s \bar{R} (\bar{\tau} \cdot \bar{n}) \mathrm{d}s \tag{8}$$

$$\bar{N}_r = 2h_0 \int_s \bar{n} \times \bar{R} (\bar{\tau} \cdot \bar{n})^2 \mathrm{d}s \tag{9}$$

where $\bar{\tau}$ equals $a_0\bar{i} + b_0\bar{j} + c_0\bar{k}$, the incident solar radiation vector; $\bar{n}$ is the outward unit normal to the elemental surface, ds; $\bar{R}$ the positive vector of the surface element, ds, relative to the center of mass; h_0 the solar energy constant $= 4.64 \times 10^{-6} N/\mathrm{m}^2$; and a_0, b_0, c_0 the direction cosines of the incident solar radiation with respect to the directions x, y, z respectively. The integration over the sunlit area s is bounded by the condition, $\bar{\tau} \cdot \bar{n} \leq 0$. The moment on a structure whose surface has an arbitrary coefficient of reflectivity ϵ_r is given by[10]

$$\bar{N}_{\epsilon r} = \bar{N}_a + \epsilon_r (\bar{N}_r - \bar{N}_a) \tag{10}$$

The moment on a beam which undergoes thermal deflection and whose surface completely absorbs all the incident radiation is obtained [after evaluating the integral in Eq. (8)] as,

$$\bar{N}_a = a_0 c_0 \delta_0 \ell h_0 \bar{j} \tag{11}$$

where the incident radiation is assumed to lie in the x, y plane ($b_0 = 0$); b is the width of the surface ($b \ll \ell$ for a thin beam); and δ_0 the maximum deflection [from Eq. (7)] $= z_{\mathrm{max}}$. The maximum deflection δ_0 can be obtained as a function of θ_i by selecting a function to represent ΔT in Fig. 4, and then using the function for ΔT in Eq. (7). The moment acting on a completely reflecting beam surface is obtained through numerical integration, as

$$\bar{N}_r = -0.05 a_0 c_0 \delta_0 \ell b h_0 \bar{j} \tag{12}$$

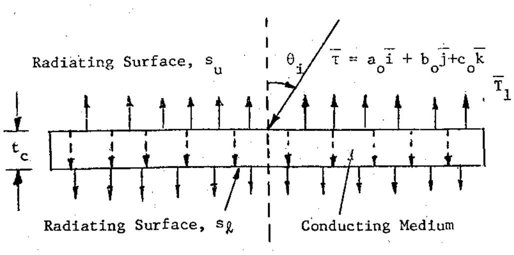

Fig. 3 Thermal gradient in a beam due to solar radiation heating.

The corresponding moment expressions for a plate are obtained as

$$\bar{N}_a = c_0 \delta_0 \ell b h_0 (b_0 \bar{i} + a_0 \bar{j})$$
$$\bar{N}_r = -0.05 c_0 \delta_0 \ell b h_0 (b_0 \bar{i} + a_0 \bar{j}) \tag{13}$$

The moment on the structural surfaces, whose coefficient of reflectivity is ϵ_r, can then be obtained by using Eq. (10) as

$$\bar{N}_{\epsilon r} = c_1 \delta_0 \ell b h_0 (b_0 \bar{i} + a_0 \bar{j}) [(1-\epsilon_r) - 0.05\epsilon_r] \quad \text{(for a plate)}$$
$$= a_0 c_0 \delta_0 \ell b h_0 \bar{j} [(1-\epsilon_r) - 0.05\epsilon_r] \quad \text{(for a beam)} \tag{14}$$

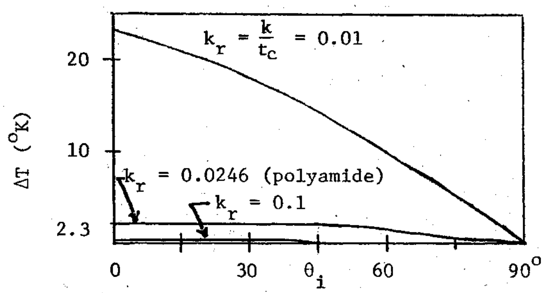

Fig. 4 Thermal gradient in a beam as a function of solar incidence angle and thermal conductivity.

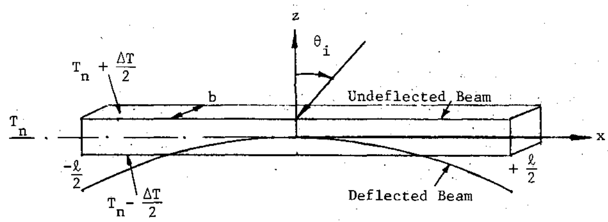

Fig. 5 Beam bending due to solar radiation heating.

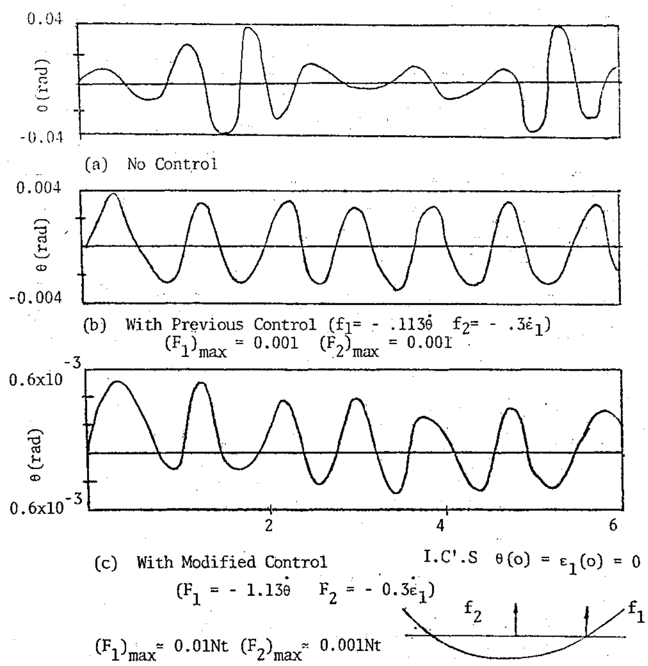

Fig. 6 Response of the beam nominally oriented along the local vertical under the influence of solar radiation disturbance caused by thermal deformation of the beam ($\delta_{th} = 0.001\ \ell$, $\ell = 100$ m).

Dynamics and Control of Beams and Plates Under the Influence of Solar Radiation Disturbances

The dynamic models of beams and plates for both cases of orbital orientations (Figs. 1 and 2) developed in Refs. 11 and 12 are considered. The nominal local horizontal orientation of beams and plates represents a gravitationally unstable motion due to the unfavorable moment of inertia distribution. Stabilizing gravity-gradient forces on such structures can be obtained by using a rigid dumbbell so that the resulting inertia distribution provides the desired gravity forces. A dumbbell may be attached to the main structure through a hinge which could provide both torsional stiffness and damping. The dynamics of the proposed dumbbell stabilized beam and plate (Figs. 1a and 2a) was considered in Ref. 12. Here, the study is extended to consider the disturbances that result from the interaction of solar radiation pressure with the thermal deformation of a structure. The disturbance resulting from the solar radiation pressure on the dumbbell will be neglected, since the dumbbell would have a small surface area compared with the main structure. The modified control laws and gain values developed in Ref. 2 and 3 will be used to obtain closed-loop transient responses of these structures by incorporating the disturbance expressions [Eq. (14)] into the structural models of the beams and plates.[1-3,11,12] The maximum thermal deflection for each case is assumed to be 0.001 ℓ based on the calculation of deflections for a 100 m long, 0.01 m thick beam made of polyamide and aluminum, respectively (see Table 1).

The beams and plates are assumed to have a fundamental frequency equal to ten times the orbital frequency with the orbital frequency corresponding to a geosynchronous orbit. Initial conditions are assumed to be zero for all the modes in order to highlight the thermally induced disturbance effect. For those cases in which the transient responses appear to be unacceptable further modifications in the control law and/or the gain values are proposed.

The Beam Along the Local Vertical

The effect of the thermally induced disturbance on the pitch motion of the beam is shown in Fig. 6(a). The disturbance (without control) has no effect on the flexible modal oscillations; hence, these are not depicted in the figure. It is seen that the pitch response has a maximum amplitude of 2.4 deg as a result of the disturbance. Application of the previously developed control law and the gain values[2] for the case of two actuators, located at the beam center and at one of the nodal points of the first symmetric mode, shows [Fig. 6(b)] pitch amplitude oscillations of less than 0.24 deg amplitude. The peak control forces required are only of the order of 10^{-3} N for each actuator. By increasing the gain value proportional to the pitch rate by a factor of ten, a further reduction of the

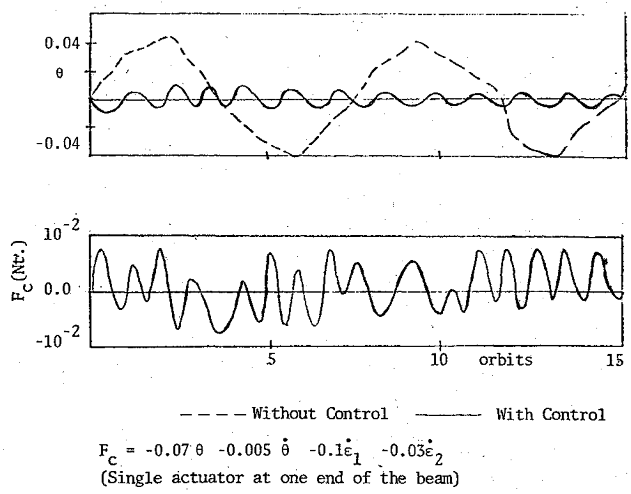

Fig. 7 Response of the dumbbell-stabilized beam under the influence of solar radiation disturbance caused by thermal deflection of the beam (δ_{th} = 0.001 ℓ, ℓ = 100 m).

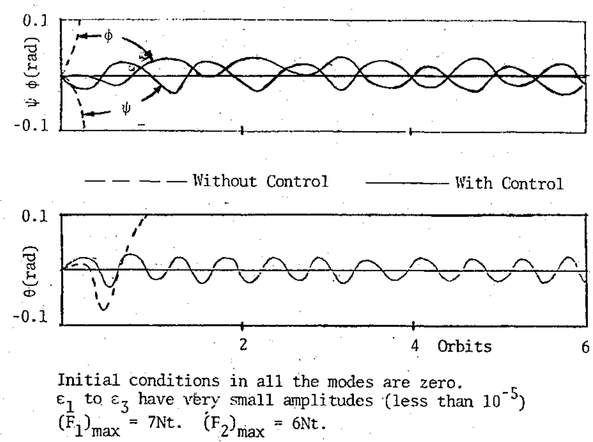

Fig. 8 Response of the plate nominally oriented along the local vertical under the influence of solar radiation disturbance caused by thermal deflection of the plate (δ_{th} = 0.001 ℓ, ℓ = 100 m). Control law based on LQR, Q = 100I, R = I.

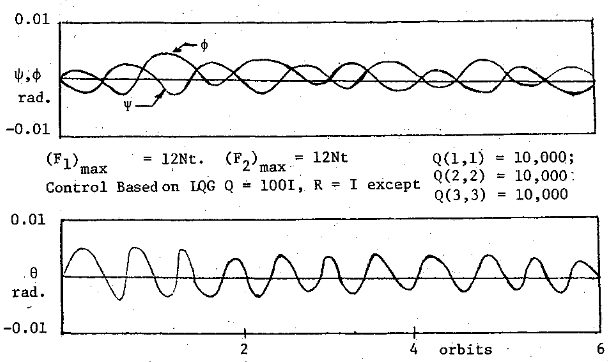

Fig. 9 Response of the plate nominally oriented along the local vertical under the influence of solar radiation disturbance caused by thermal deflection of the plate (δ_{th} = 0.001 ℓ) ℓ = 100 m, ω_I = 10.

Table 1 Properties of representative materials[9]

Material	Density, kg/m³	Expansion coefficient, α_e, m/m°C	Thermal conductivity K, kW/m-K	≃Cost, $/kg	δ_{max},[a] m
Graphite	1.5×10^3	8.3×10^{-5}	8.65×10^{-3}	500	10^{-4}
Beryllium	1.8×10^3	3.5×10^{-6}	12.25×10^{-3}	10,000	10^{-4}
Aluminum	2.7×10^3	2.1×10^{-6}	28.8×10^{-3}	1.1	10^{-5}
Polyamide	1.13×10^3	25×10^{-6}	2.45×10^{-3}	15	7

[a]δ_{max} = maximum thermal deflection of a plate with sides equal to 100 m and thickness equal to 0.01 m.

pitch amplitude to approximately 0.03 deg is illustrated [Fig. 6(c)]. Correspondingly, the peak force requirement in actuator number one increases to 0.01 N.

The Dumbbell-Stabilized Beam

The transient response of the dumbbell-stabilized beam due to solar radiation pressure which induces thermal deformation on a beam is shown in Fig. 7 (only the pitch mode is depicted). The pitch oscillations are seen to have approximately 2.4 deg amplitudes in the absence of control. With the application of the control law previously developed in Ref. 3 (and indicated in Fig. 7), the amplitude of the pitch oscillations is reduced to 0.24 deg. The peak control force required is about 0.01 N. The gain values can easily be modified further to meet any specific requirement on the pitch motion of the beam.

The Plate Oriented Along the Local Vertical

The uncontrolled and the controlled transient responses of the 100 m square thin plate nominally oriented along the local vertical and with the disturbance caused by the thermal deflection of the plate are shown in Fig. 8. The same order of magnitude thermal deflections are assumed here as for the beam in the two previous sections. The pitch, roll and yaw rotations of the plate exceed the linear range in less than an orbit. Application of the linear quadratic Gaussian control technique with the penalty matrices, $Q=100I$ and $R=I$, results in transient responses in which steady state oscillations with amplitudes of about 0.02 rad are seen in all three rotational modes of the plate (see Fig. 8). Modal oscillations (nondimensionalized) in all the three flexible modes remain within an amplitude of 10^{-5} (10^{-3}m). An improvement in the transient response was obtained by employing a split weighting state penalty matrix [$Q=100\ I$, except for $Q(1,1)=Q(2,2)=Q(3,3)=10{,}000$] where the rotational modes were penalized more heavily. The transient response of the plate in the rotational modes for this case is shown in Fig. 9. The steady state oscillations are reduced by an order of magnitude in comparison with Fig. 8. However, the peak control forces increased from 7 to 12 N. The total control effort required also increased by approximately 60%.

The Dumbbell Stabilized Plate

The closed-loop transient response of the dumbbell-stabilized plate is considered. The magnitude of the pitch, roll and yaw angles are seen to be within 0.02 rad in the absence of any solar radiation pressure induced disturbance [Fig. 10(a)]. For the same case, the effect of the solar radiation pressure disturbance that results from thermal deformations of the plate ($\delta_{th}=0.001\ell$, $\ell=100m$) is shown in Fig. 10(b). The pitch and the yaw oscillations are seen to exceed the linear range even with the control. The control effort required (3×10^5 N-s) was nearly ten times more than that for the case without the disturbance [Fig. 10(a)]. The transient response characteristics for this case are therefore unacceptable.

A redesign of the control is attempted with penalty matrices selected as: $Q=10{,}000I$ and $R=100I$. (Both the state as well as the control are now penalized more heavily by increasing both sets of elements by two orders of magnitude.) The transient response of the dumbbell stabilized plate with this control is shown in Fig. 11. The pitch, roll and yaw amplitudes are well within 0.02 rad even in the presence of the disturbance. The peak control force required is approximately 14 N in both actuators (or an rms value of a little less than 30 N).

Thus, the thermal deformations of the structures can be of greater concern than the deformations of the structure due to structural vibrations (considered in Refs. 2 and 3) in modeling the disturbances arising from the solar radiation pressure. This study shows the need to further minimize thermal deformations ($\ll 0.001\ \ell$) from the view point of reducing the radiation pressure disturbance effects. This can be accomplished within cost and strength constraints primarily by increasing the thermal conductivity.

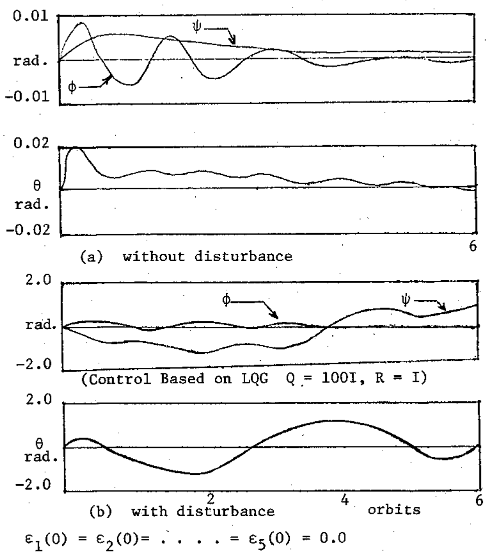

Fig. 10 Response of the dumbbell-stabilized plate under the influence of solar radiation disturbance caused by thermal deflection of the plate ($\delta_{th}=0.001\ \ell$) $\ell=100$ m, $\omega_I=10$.

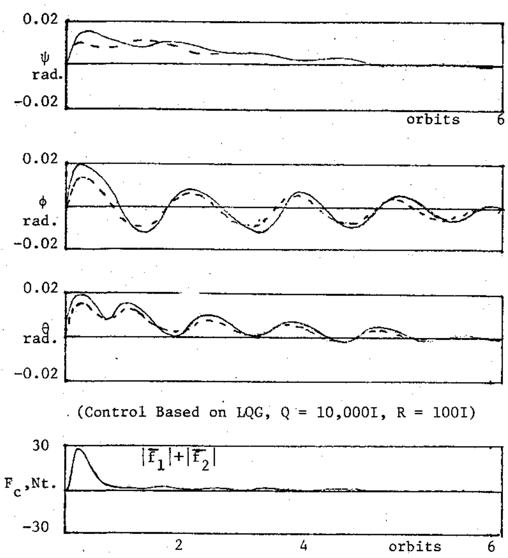

Fig. 11 Response of the dumbbell-stabilized plate under the influence of solar radiation disturbance caused by thermal deflection of the plate ($\delta_{th}=0.001\ \ell$) $\ell=100$ m, $\omega_I=10$.

Conclusion

The dynamics and control of orbiting beams and plates that interact with solar radiation pressure are studied. The major effect of the solar radiation pressure is found to result in net moments on the structure. Modifications of control laws and/or feedback gain values previously obtained by not considering the thermal disturbances are suggested to improve the transient response characteristics from the effects of thermal induction. In general, the effect of solar radiation pressure acting on structures which undergo static thermal deformation is found to be more important than the effect of solar radiation pressure on vibrating structures which experience no thermal deformation. In order to reduce the disturbances resulting from the interaction of solar radiation pressure with a structure affected by thermal deformation, further minimization of the thermal deformations is recommended.

Acknowledgment

Research was supported by NASA Grant NSG-1414, Supplement 6 and the NASA/Howard University Large Space Structures Institute.

References

[1]Krishna R. and Bainum, P.M., "Effect of Solar Radiation Disturbance on a Flexible Beam in Orbit," *AIAA Journal*, Vol. 22, May 1984, pp. 677-682.

[2]Krishna, R. and Bainum, P.M., "Orientation and Shape Control of an Orbiting Flexible Beam Under the Influence of Solar Radiation Pressure," AIAA Paper 83-325, Jan. 1983; see also *Astrodynamics 1983, Advances in the Astronautical Sciences,* Vol. 54, Part I, pp. 221-238.

[3]Bainum, P.M. and Krishna, R., "Control of an Orbiting Flexible Platform in the Presence of Solar Radiation," *Proceedings of Fourteenth International Symposium on Space Technology and Science,* Tokyo, 1984, pp. 937-944.

[4]Frisch, H.P., "Thermally Induced Response of Flexible Structures: A Method for Analysis," *Journal of Guidance and Control,* Vol. 3, Jan.-Feb. 1980, pp. 92-94.

[5]Ayer, F., and Soosar, K., "Structural Distortions of Space Systems Due to Environmental Disturbances," AIAA Paper 80-0854, June 1980.

[6]Shu, C.F., and Chang, M.H., "Integrated Thermal Distortion Analysis for Satellite Antenna Reflectors," AIAA Paper 84-0142, Jan. 1984.

[7]Gray, W.A. and Muller, R., *Engineering Calculations in Radiative Heat Transfer,* Pergamon Press, New York, 1974.

[8]Burgreen, D., *Elements of Thermal Stress Analysis,* C.B. Press, New York, 1971.

[9]Ashton, J.E., Halpin, J.C., and Petit, P.H., *Primer on Composite Materials Analysis,* Technomic Publishing Co., Lancaster, Pa., 1970.

[10]Karymov, A.A., "Determination of Forces and Moments Due to Light Pressure Acting on a Body in Motion in Cosmic Space," *Prikladnaia Matematica I Mechanika,* Vol. 26, No. 5, 1962, pp. 867-876.

[11]Kumar, V.K. and Bainum, P.M., "Dynamics of a Flexible Body in Orbit," *Journal of Guidance and Control,* Vol. 3, Jan.-Feb. 1980, pp. 90-92, see also AIAA Preprint No. 78-1418.

[12]Bainum, P.M. and Kumar, V.K., "On the Dynamics of Large Orbiting Flexible Beams and Platforms Oriented Along the Local Horizontal," *Acta Astronautica,* Vol. 9, No. 3, 1982, pp. 119-127.